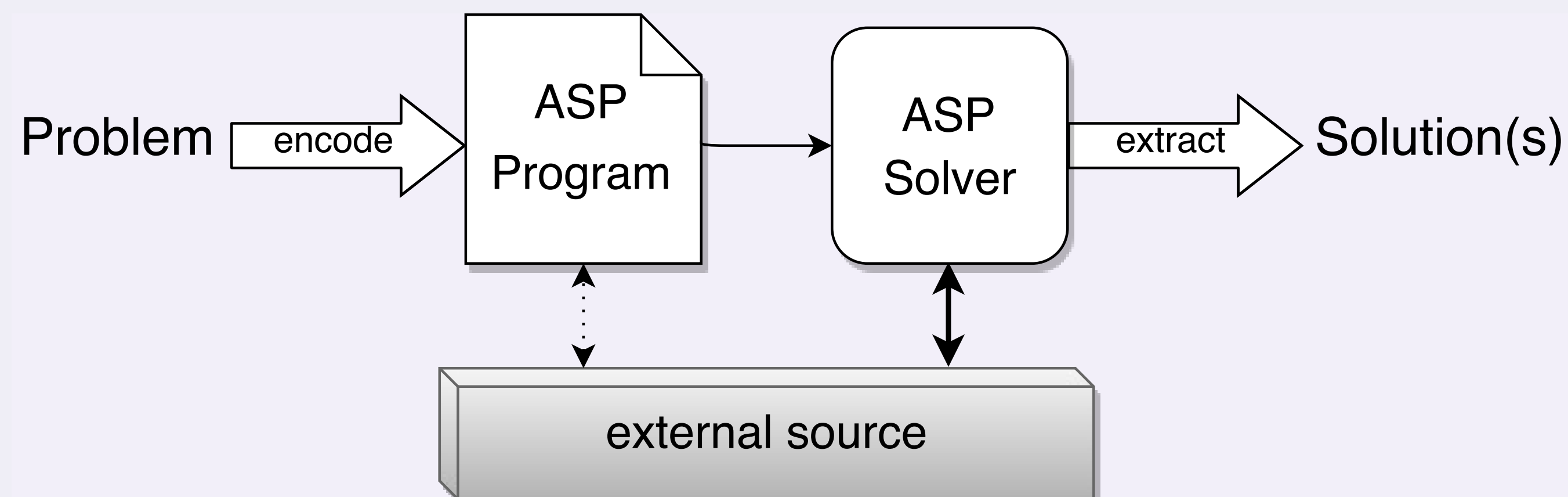


# Exploiting Partial Assignments for Efficient Evaluation of Answer Set Programs with External Source Access

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## 1. Motivation



- ▶ HEX-programs extend ASP by **external sources**
- ▶ Similar to SMT for SAT, but external source is **black box**
- ▶ Rule bodies may contain **external atoms** of the form  $\&g[p](c)$ 
  - ▶  $g$  is an external predicate name
  - ▶  $p = p_1, \dots, p_k$  are input predicate names or constants
  - ▶  $c = c_1, \dots, c_l$  are output terms

**Semantics:** Boolean oracle function  $f_{\&g}$  s.t.  $\&g[p](c)$  is true iff  $f_{\&g}(A, p, c)$ , w.r.t. assignment  $A$

- ▶ **Basic evaluation:**
  1. replace  $\&g[p](c)$  by  $e_{\&g[p]}(c)$ , add  $e_{\&g[p]}(c) \vee ne_{\&g[p]}(c)$
  2. run CDNL solver (e.g. Clasp)
  3. check guess for  $\&g[p](c)$  when  $p$  decided
  4. learn io-nogoods when evaluating external atoms to avoid wrong guesses

**Challenge:** External sources cannot guide the solver effectively, they are **black boxes** evaluated under **complete** assignments!

## Example

Oracle function for checking if size of predicate extension  $\geq n$ :

$$f_{\&geq}(A, p, n) = \begin{cases} \mathbf{T} & \text{if } |\{Tp(x, y) \in A\}| \geq n \\ \mathbf{F} & \text{otherwise} \end{cases}$$

HEX-program:

```
vertex(a). vertex(b).
a(X, Y) v na(X, Y) ← vertex(X), vertex(Y).
                    ← e_{\&geq[a,2]}().
e_{\&geq[a,2]}() v ne_{\&geq[a,2]}() ←
```

$A : \{\mathbf{F}e_{\&geq[a,2]}(), \mathbf{T}a(a, b), \mathbf{F}a(b, a), \mathbf{T}a(a, a), \mathbf{F}a(b, b)\}$   
Learn :  $\{\mathbf{F}e_{\&geq[a,2]}(), \mathbf{T}a(a, b), \mathbf{F}a(b, a), \mathbf{T}a(a, a), \mathbf{F}a(b, b)\}$

## 2. Main Contributions

Extension from two-valued to **three-valued assignments**, enables:

1. **Early evaluation** of external sources
2. External theory learning producing **smaller nogoods**
3. Nogood **minimization** techniques

New techniques applicable by user **without expert knowledge**  
Benchmarks show **effectiveness of techniques**

## References

- Thomas Eiter, Giovambattista Ianni, Roman Schindlauer, and Hans Tompits: "A Uniform Integration of Higher-Order Reasoning and External Evaluations in Answer-Set Programming", IJCAI, 2005.  
Thomas Eiter, Michael Fink, Thomas Krennwallner, and Christoph Redl: "Conflict-driven ASP solving with external sources", TPLP, 2012.  
Robert Nieuwenhuis, Albert Oliveras, and Cesare Tinelli: "Solving SAT and SAT Modulo Theories: From an abstract Davis-Putnam-Logemann-Loveland procedure to DPLL(T)", ACM Journal, 2006.

## 3. Extension to Partial Assignments

**Partial assignment** over atoms  $\mathcal{A}$  is set  $A$  of signed literals  $\mathbf{T}a$ ,  $\mathbf{F}a$  and  $\mathbf{U}a$  s.t. for all  $a \in \mathcal{A}$  exactly one of  $\mathbf{T}a \in A$ ,  $\mathbf{F}a \in A$  or  $\mathbf{U}a \in A$  holds.

A **three-valued oracle function**  $f_{\&g}$  for  $\&g[p](c)$  is a function such that  $f_{\&g}(A, p, c) \in \{\mathbf{T}, \mathbf{F}, \mathbf{U}\}$  for a partial assignment  $A$  and all possible values of  $p$  and  $c$ .

A three-valued oracle function  $f_{\&g}$  is **assignment-monotonic** if  $f_{\&g}(A, p, c) = X$ ,  $X \in \{\mathbf{T}, \mathbf{F}\}$ , implies  $f_{\&g}(A', p, c) = X$  for all assignments  $A' \succeq A$ .

## 4. Nogood Learning with Partial Assignments

**Nogood learning:** Nogood only containing the decided part of a partial assignment learned as soon as oracle function evaluates to  $\mathbf{T}$  or  $\mathbf{F}$   
Partial nogoods often **significantly smaller**

**Nogood minimization:** Given an io-nogood  $N$ , its minimized nogoods are

$$\text{minimize}(N) = \{N' \subseteq N \mid N' \text{ is an io-nogood, } f_{\&g}(N'', p, c) = \mathbf{U} \text{ for all } N'' \subsetneq N'\}.$$

Nogoods with same input part can be minimized **simultaneously**

## Example

Extension to three-valued oracle function:

$$f_{\&geq}(A, arc, n) = \begin{cases} \mathbf{T} & \text{if } |\{\mathbf{T}arc(X, Y) \in A\}| \geq n \\ \mathbf{U} & \text{if } |\{\mathbf{T}arc(X, Y), \mathbf{U}arc(X, Y) \in A\}| \geq n \\ \mathbf{F} & \text{otherwise} \end{cases}$$

External source can already be checked under partial assignment:

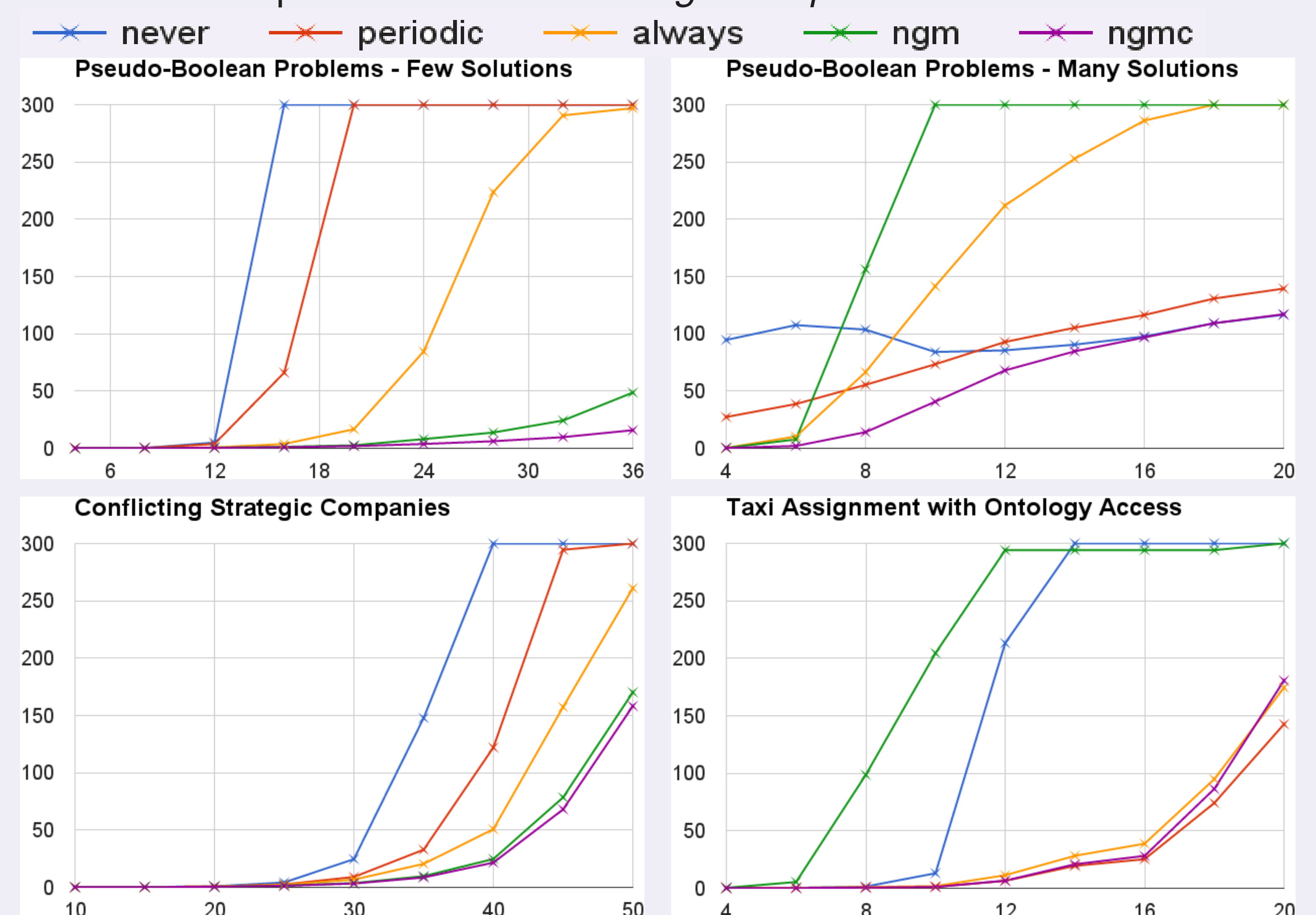
$A : \{\mathbf{F}e_{\&geq[a,2]}(), \mathbf{T}a(a, b), \mathbf{F}a(b, a), \mathbf{T}a(a, a), \mathbf{U}a(b, b)\}$

Learn :  $\{\mathbf{F}e_{\&geq[a,2]}(), \mathbf{T}a(a, b), \mathbf{F}a(b, a), \mathbf{T}a(a, a)\}$

Learn minimal :  $\{\mathbf{F}e_{\&geq[a,2]}(), \mathbf{T}a(a, b), \mathbf{T}a(a, a)\}$

## 5. Empirical Evaluation

Use *Divhex* implementation with *Gringo/Clasp* as backends



average runtime over 50 instances (sec.) vs. instance size, (timeout 300 sec.)

- ▶ **Significant improvements** if not very many answer sets
- ▶ **Tradeoff:** time for evaluating external atom  $\leftrightarrow$  information gain
- ▶ Benefit of **nogood minimization** depends on size of nogoods